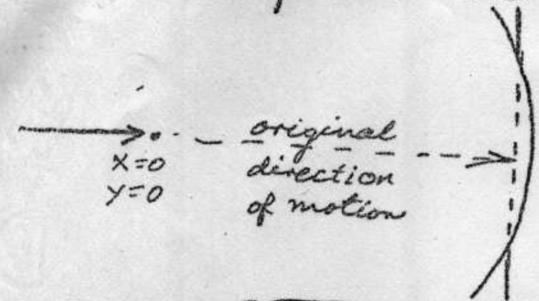
ELECTRODYNAMICS

Notes, Wed 6 Apr '77

Notes about the problem of the suddenly-accelerated-sideways particle and the radiation from it (to supplement the self-tutorial of last week). (1) "Information" from the origin at laboratory time t after the sudden deflection has got to a distance described by a sphere of radius $\boxed{r = ct}$

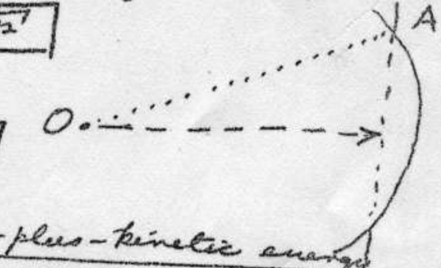


(2) The electron itself in the same time, (if undeflected, would) have got to a distance $x = \beta ct$.

(3) The straight line $x = \beta ct$ intersects the circle $x^2 + y^2 = r^2 = c^2t^2$ at a point with x-coordinate $\boxed{x = \beta ct}$ and y-coordinate $\boxed{y = \sqrt{c^2t^2 - x^2} = \sqrt{c^2t^2 - c^2t^2\beta^2} = ct\sqrt{1-\beta^2}}$

(4) The slope of the line OA is

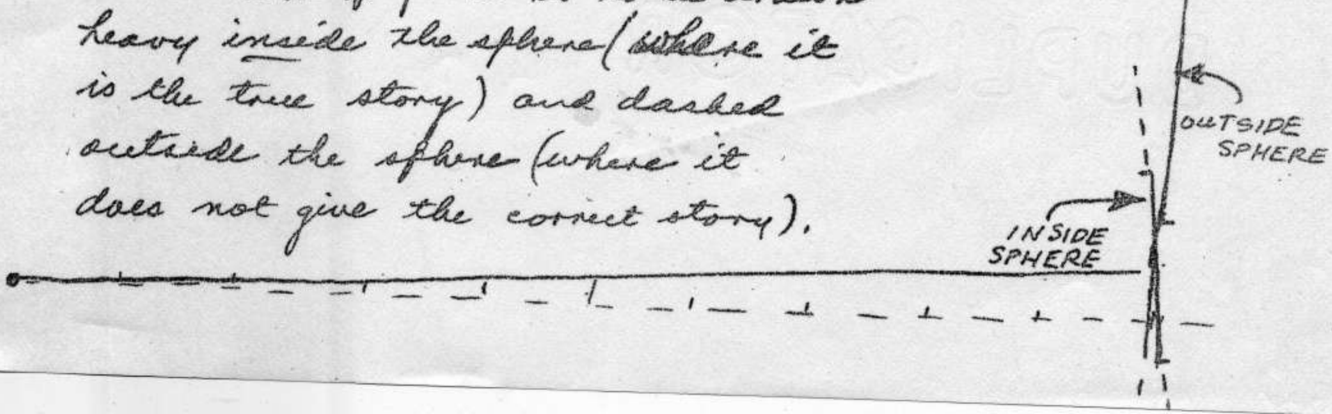
$$\boxed{\text{slope} = \frac{y\text{-coord. of } A}{x\text{-coord of } A} = \frac{ct\sqrt{1-\beta^2}}{ct\beta} = \frac{\sqrt{1-\beta^2}}{\beta}}$$



(5) For a particle with $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{\text{rest-plus-kinetic energy}}{\text{rest energy}} = \frac{E}{mc^2} = 10$ we have $\sqrt{1-\beta^2} = \frac{1}{10}$, $1-\beta^2 = \frac{1}{100}$, $\beta^2 = \frac{99}{100}$, $\beta = \frac{99.5}{100}$

Thus for such a fast particle the slope of OA is $\boxed{\text{slope} = \frac{\sqrt{1-\beta^2}}{\beta} = \frac{\left(\frac{1}{10}\right)}{\left(\frac{9.95}{10}\right)} = \frac{1}{9.95} \sim \frac{1}{10}}$, so $\boxed{\text{also angle is} \sim \frac{1}{10}}$.

(6) We are considering an angle of deflection of $\frac{1}{20}$ radian, no change of velocity. So we take whole drawing above and tilt it up (or down) by $\frac{1}{20}$ radian to get story of "central line of force" accompanying particle after deflection, except now we recognize that the "central line of force" is to be drawn heavy inside the sphere (where it is the true story) and dashed outside the sphere (where it does not give the correct story).



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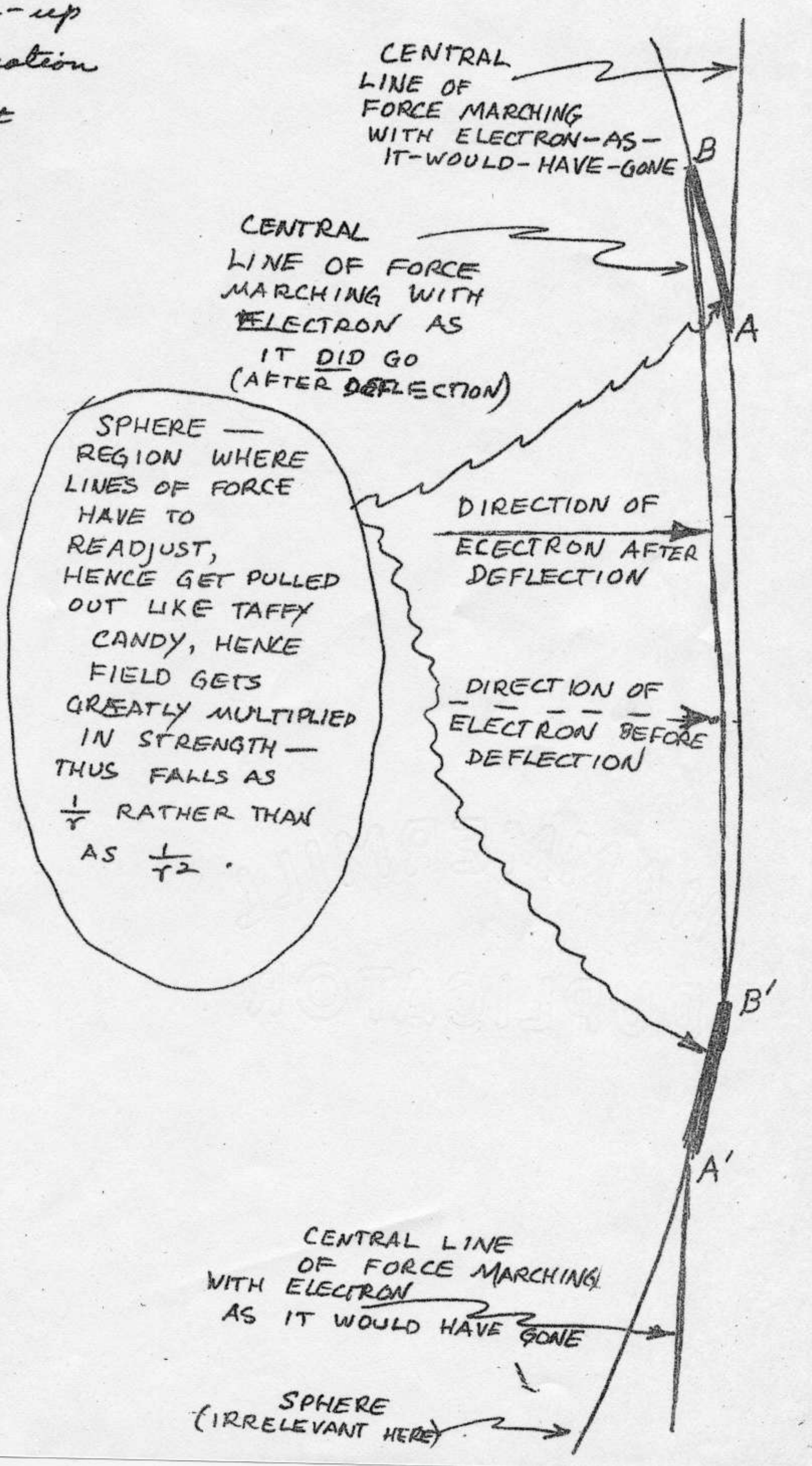
(7) On a blown-up scale, the situation is as shown at the right.



(8) The region of strong radiation, insofar as we judge from the story of "the central line of force", as depicted on the previous page, extends from A to B, and from A′ to B′.

from angle $\theta_{rad} \sim \frac{1}{\gamma}$ (at A)

to $\theta_{rad} \sim \frac{1}{\gamma} + \theta_{defl}$ (at B) ← angle of deflection

from angle $\theta_{rad} \sim -\frac{1}{\gamma}$ (at A′)

to $\theta_{rad} \sim -\frac{1}{\gamma} + \theta_{defl}$ (at B′)

(I give these details here because there was not room to give them in the last parts of my comments on the self-test self-tutorial handed in on Friday Apr 1 and now soon to be given back. I am impressed by how far into the situation, and on the whole how well, most of us got in the analysis of the radiation pattern, especially with a set of questions being tried out on that self-tutorial for the first time —— and therefore not fully "debugged". The greatest need was a magnified scale — hence the new drawing on page 2 of these notes)

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"REGIME ANALYSIS"

In Problem 14.14 one found that the average power output of synchrotron over a 1 period speedup from rest has a frequency distribution

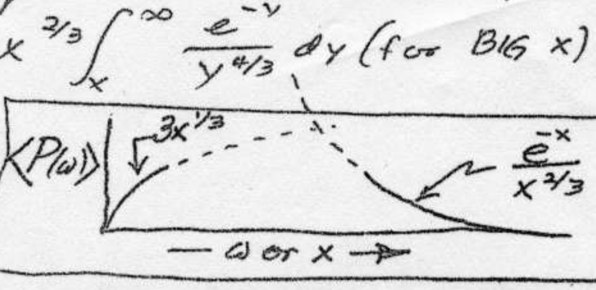
$$\langle P(\omega,t)\rangle = \frac{2^{2/3}}{3\pi}\frac{e^2}{\rho}\gamma_{max}\cdot\boxed{x^{2/3}\int_x^\infty \frac{e^{-y}}{y^{4/3}}\,dy} \qquad \boxed{\text{Here } x=\frac{2\omega}{\omega_{c,max}}}$$

and it was asked,
what is the limiting
form of this expression
for low and high freq. This is the key factor.

Low freq., $\boxed{x \text{ SMALL}}$ Nothing could be worse than to say let x go to zero and call $\int_0^\infty \frac{e^{-y}}{y^{4/3}}\,dy = \Gamma(-\frac{1}{3})$ because $\Gamma(-\frac{1}{3})$ is finite and the integral is <u>infinite</u>.

Instead, note all the contrib'n comes from near $y=x$, that is, for <u>very small</u> y. So in the region of substantial contribution e^{-y} is essentially unity. So we deal with $x^{2/3}\int_x^\infty \frac{dy}{y^{4/3}}$
$= x^{2/3}\left(\frac{3}{x^{1/3}}\right) = 3x^{1/3}$

High freq., $\boxed{x \text{ BIG}}$. As y goes from $x=10$, for example, to $x=11$, the factor $\frac{1}{y^{4/3}}$ does not change percentagewise very much, so we call it $\frac{1}{x^{4/3}}$. The factor that <u>does</u> change is e^{-y} (drops by factor $\frac{1}{e}=\frac{1}{2.718}$ from $y=10$ to $y=11$)
So write $x^{2/3}\int_x^\infty \frac{e^{-y}}{y^{4/3}}\,dy$ (for BIG x) as $\sim \frac{x^{2/3}}{x^{4/3}}\int_x^\infty e^{-y}\,dy$
or $\frac{e^{-x}}{x^{2/3}}$.



It's clear from these 2 limits that the fn. <u>peaks</u> in between