ELECTRODYNAMICS Wed. 2 Feb 1977 (1)

Problems due 10 am Friday 4 Feb. 1977

Jackson 11.9, 11.19, 12.6, 12.7

Discussion. The first problem is connected with, and a part of, one way of analyzing the Thomas precession. In that connection it is useful to refer back to the notes of 24 Jan., p. 10, where Eq. (41.43) gives the whole story of the Thomas precession in brief,

$$R(\omega dt)\, B^{-1}(a_{comoving}\, dt) = B^{-1}(\beta + d\beta)\, B(\beta) \tag{41.43}$$

"WHATEVER THE PRICE.... I HAD TO BRING ABOUT A POSITIVE RESULT".

Quotes from a 1931 letter from Max Planck to JAW's Professor Robert W. Wood, Johns Hopkins University: "Dear Colleague ...... Briefly put, I can describe the whole effort as an act of desperation, for by nature I am peaceful and against dubious adventures. But I had been fighting for six years from 1894 on with the problem of the equilibrium between radiation and matter without having any success. I knew that this problem is of fundamental significance for physics. I knew the formula which provides the energy distribution in the normal spectrum. A theoretical explanation, therefore, had to be found at all costs, whatever the price......... No matter what the circumstances, may it cost what it will, I had to bring about a positive result." [Planck's life time of studies had led toward the place of mystery. That place once revealed, it took him two weeks of the most strenuous work in his life to uncover the magic concept. To clarifying and consolidating that concept he dedicated the rest of his life].

It is important to know (1) what this formula is all about (2) what each successive term in it means (3) what are the knowns and what are the unknowns (4) the key result,

$$\omega\, dt = [2 \sinh^2(\alpha/2)]\, dn_\alpha \times n_\alpha$$

or for low velocities

$$\omega = \underset{\text{acceleration}}{a} \times \underset{\text{velocity}}{\beta}/2$$

Thus only the component of the acceleration perpendicular to the velocity is relevant for the Thomas precession.

Re problem 11.19, recall the especially beautiful and simple nature of a Lorentz transformation for a photon proceeding in the direction of, or opposite to the direction of, a boost. In general, for boost in x-direction,

$$E = E' \cosh\alpha + p^{x'} \sinh\alpha$$
$$p^x = E' \sinh\alpha + p^{x'} \cosh\alpha$$
$$p^y = p^{y'}$$
$$p^z = p^{z'}$$

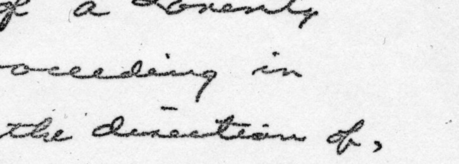
but when photon is going in x-direction, $p^{x'} = E'$ and

$$p^x = E = E'\cosh\alpha + E'\sinh\alpha = e^{\alpha} E' = e^{\alpha} p^{x'}$$

Similarly, for photon going in negative x direction, $p^{x'} = -E'$ and we have

$$E = e^{-\alpha} E'$$

What about the product of the energies of photons proceeding in opposite directions?



To produce an (e^+e^-) pair <u>at rest</u> (the "cheapest" possible production) one needs a <u>total energy of $2mc^2$</u> (small correction neglected if e^+e^- are bound in lowest state of a "light hydrogen atom" — its binding is $\frac{1}{2}\,13.6 = 6.8\,eV$, compared to $2mc^2 = 2 \times 0.511\,MeV = 1.022\,MeV$)

and a <u>total momentum of zero</u>. Latter implies that the two impacting photons have momentum of equal magnitudes and opposite directions. Conclude one photon has

$$E_I = p_I^x = +mc^2$$

other has $E_{II} = -p_{II}^x = +mc^2$

As seen in a boosted frame

one has $E_I = mc^2 e^{\alpha}$

other has $E_{II} = mc^2 e^{-\alpha}$

<u>Re</u> <u>problem 12.6</u>: **Notes** that the easy way to get on top of the situation is to recall that $(\underset{\sim}{E} \cdot \underset{\sim}{B})$ and $(\underset{\sim}{E}^2 - \underset{\sim}{B}^2)$ have the same values in all Lorentz frames that are "simply connected" with the original frame — i.e., no reflections

Note also that a boost in the direction $\underset{\sim}{n}$ (UNIT VECTOR)

of $\underset{\sim}{E} \times \underset{\sim}{B}$ with velocity parameter $\underline{\underline{\alpha}}$ (NOT 2α!)

will give zero Poynting flux (i.e., will

give $\underset{\sim}{E}' \times \underset{\sim}{B}' = 0$, or $\underset{\sim}{E}' \Uparrow \underset{\sim}{B}'$ parallel)

when α is given the value such that

$$\underset{\sim}{n} \tanh 2\alpha = \frac{\text{Poynting flux}}{\text{Energy density}} = \frac{\left(\frac{E \times B}{4\pi}\right)}{\left(\frac{E^2 + B^2}{8\pi}\right)}$$

↑ UNIT VECTOR

Re problem 12.7 it makes the path to the solution quicker, and provides some extra insight along the way, to take the motion listed by Jackson as known, and from it figure out the field!

$$m \frac{d^2 x^\alpha}{d\tau^2} = e F^{\alpha\beta} \frac{dx_\beta}{d\tau}$$

Regard these as given. Find $F^{\alpha\beta}$!

$$x = A\left(\frac{mc^2}{eB}\right) \sin\left(\frac{eB\tau}{mc^2}\right)$$

$$y = A\left(\frac{mc^2}{eB}\right) \cos\left(\frac{eB\tau}{mc^2}\right)$$

$$z = \sqrt{1 + A^2}\left(\frac{mc^2}{eE}\right) \cosh\left(\frac{eE\tau}{mc^2}\right)$$

$$t = \sqrt{1 + A^2}\left(\frac{mc^2}{eE}\right) \sinh\left(\frac{eE\tau}{mc^2}\right)$$

Skip all of Jackson's fancy "R" and "ρ" notation and "stick to basics" that is, e, B, mc^2, E, and τ.

At end explain how you'd go about orienting axes to get such simple results!

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Notes on previous problems

Jackson 9.26. Optical theorem says

$$\underbrace{\sigma_{scatt} + \sigma_{absn}}_{\text{dimensions cm}^2} = \underbrace{\frac{4\pi}{k}}_{\text{cm}} \underbrace{\left(\begin{array}{c}\text{Imaginary part of} \\ \text{forward scattering amp}\end{array}\right)}_{\text{cm}}$$

We recalled (from electrostatics) that a small dielectric sphere in an external field gave rise to a dipole moment equal to the strength of the external field times $\frac{\varepsilon - 1}{\varepsilon + 2} a^3$.

We also recalled that we have a dynamic situation — our small sphere is in a time-varying field and there is conductivity. Thus, in writing

$$\nabla \times B = \frac{1}{c}\frac{\partial D}{\partial t} + \frac{4\pi}{c} J$$

$$= \frac{1}{c}\left[\varepsilon \cdot -i\omega E + 4\pi\sigma E\right]$$

$$= \frac{-i\omega E}{c}\left[\varepsilon + \frac{4\pi i \sigma}{\omega}\right]$$

we see that the effect of the conductivity is most easily taken into account by replacing ε by $\varepsilon + \frac{4\pi i \sigma}{\omega}$

Forward scattering amplitude obtained by multiplying dipole moment by $\frac{\omega^2}{c^2}$ is

$$f(0) = \frac{\varepsilon - 1 + \frac{4\pi i \sigma}{\omega}}{\varepsilon + 2 + \frac{4\pi i \sigma}{\omega}} \, a^3 \, \frac{\omega^2}{c^2}$$

Imaginary part of this times $\frac{4\pi}{k}$ should give $\sigma_{scatt} + \sigma_{absn}$

Actually it only gives σ_{absn}.

Reason accuracy of soln. of $f(0)$ is not great enough ("2nd order") to yield σ_{scatt}