UNIVERSITY OF TEXAS AT AUSTIN
Department of Physics

NAME _____

ELECTRODYNAMICS: PHYSICS 387L
FINAL EXAMINATION

Friday, 13 May 1977

page one of thirteen pages

PLEASE COUNT AND VERIFY THAT YOU HAVE ALL 13 PAGES OF OUR FRIDAY THE 13th EXAMINATION BEFORE STARTING! (LUCK!)

1. LORENTZ	☐
2. SYNCHROTRON SOURCE	☐
3. SINGING SPHERE	☐
4. POTPOURRI	☐
5. VARIATIONAL PRINCIPLE	☐

PLEASE CHECK HERE THE QUESTIONS TO BE COUNTED ON THIS EXAMINATION.

--Answer three of the five questions. (No credit for more.)

--Use large, clear, well-labelled diagrams where appropriate.

--If time or memory do not suffice to answer a chosen question, outline the principles you would have used to answer it if time had sufficed.

--Turn in one copy of exam with your answer written on blank spaces (and back of sheet if necessary) and keep the other copy for your records (and for any scratch work).

--Do not use any computer. Simply INDICATE ANSWER. Example:

$$\frac{4.18 \times 10^{7} \times (2.998 \times 10^{10})^{2}}{1.602 \times 10^{-12}}$$

$\sim 2 \times 10^{40}$ eV/km^2

Do not give more than ONE DIGIT; and you will not be held even to its precise value; that is, 3 is acceptable even when the correct digit is 9.

Correct EXPONENT is essential.

No numerical answer is complete that does not give UNITS.

--"The difference between the right word and the almost right word is the difference between lightning and a lightning bug."--Mark Twain

--Please write your name at the top of each page of your work.

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I. LORENTZ

The rocket frame moves along the x-axis relative to the laboratory frame with velocity $\beta = \tanh\alpha$, so that

$t = t' \cosh\alpha + x' \sinh\alpha$, $\quad y' = y$,

$x = t' \sinh\alpha + x' \cosh\alpha$, $\quad z' = z$.

Fill in here the coefficients relating

(a) number density of particles, n (number per cm^3) and flow rate of particles, S (number of particles per cm^2 of area and per cm of light-travel time) in the two frames:

$n = n'$ [.....] [sign] S'_x [.....] ; $S'_y =$ [.....] ;

$S'_x = n'$ [.....] [ ] S'_x [.....] ; $S'_z =$ [.....] .

(b) electric current density, $\hat{J}$ in geometric time units (e.s.u. of charge per cm^2 and per cm of light-travel time) and charge density, ρ (e.s.u. of charge per cm^3) in the two frames:

$\rho =$ [.....] ; $J'_y =$ [.....] ;

$\hat{J}_x =$ [.....] ; $\hat{J}'_z =$ [.....] .

(c) electric and magnetic fields parallel ($E_{\parallel}$, $B_{\parallel}$) and perpendicular ($E_{\perp}$, $B_{\perp}$) to the direction of motion $\underset{\sim}{n} = (0, 0, 1)$ of the rocket frame relative the laboratory frame:

$E_x =$	$E'_x +$	;
$B_x =$	$B'_x +$	;
$E_y =$	E'_y	B' (sign? subscript? coeff.?);
$E_z =$	E'_z	B' " " " ;
$B_y =$	B'_y	E' " " " ;
$B_z =$	B'_z	E' " " " .

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(d) the x-component of the Poynting flux, or for that matter the (01)-component T^{01} of any symmetric tensor in terms of the components of that tensor (here the stress-energy tensor) in the rocket frame:

$$T^{01} = \cosh\alpha \sinh\alpha\, T'^{00} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots T'^{01}$$
$$+ \ldots\ldots\ldots\ldots T'^{02} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots T'^{03}$$
$$+ \ldots\ldots\ldots\ldots T'^{10} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots T'^{11}$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(e) How (outline plan only, not the details) could you use the result of (d) and a corresponding formula for T^{00} to derive the formula

$$\underline{n} \tanh 2\alpha = \frac{2\underline{E} \times \underline{B}}{\underline{E}^2 + \underline{B}^2}$$

for the Lorentz transormation

$$\underline{\beta} = \underline{n} \tanh\alpha$$

thatreduces the Poynting flux to zero?

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II. SYNCHROTRON RADIATION SLOWS SOURCE.

A particle of rest mass m and charge e is moving with a relativistic rest-plus-kinetic energy $E = mc^2 \cosh\alpha$ at right angles to a uniform static magnetic induction B.

(a) In terms of e, m, c, α (or $\gamma = \cosh\alpha$) and B how much is the energy radiated per sec in the laboratory frame?

(b) Show that the energy at any time t is connected with the original energy by the formula

$$(E/mc^2) = \coth\,[(2e^4B^2/3m^3c^6)\,(ct + \text{const})]$$

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II. (b) work space continued

(c) When a particle is trapped in the magnetic dipole field of the earth and is spiraling north and south along a magnetic line of force, does it radiate more energy per sec while near the equator than it does while near its turning points? Why? Make quantitative statements.

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III. THE SINGING SPHERE

A perfectly conducting sphere of radius $\underline{a}$, though it has been RADIATING FOR A LONG TIME, is still emitting a pure outgoing $\ell = 3$ TM electromagnetic wave, the amplitude of which as picked up at a distant but fixed point of reception in these LATE TIMES varies with time as

$$e^{-i\omega t} = e^{-i\omega_0 t - (\Gamma/2)t}.$$

Remembering that

$$\underset{\sim}{B}^{(E)}_{\ell m} = f_\ell(kr)\,\underset{\sim}{L}\,Y_{\ell m}(\theta,\varphi)$$

$$= \underset{\sim}{L}\,f_\ell(kr)\,Y_{\ell m}(\theta,\varphi)$$

and

$$\underset{\sim}{E}^{(E)}_{\ell m} = (i/k)\,\underset{\sim}{\nabla} \times \underset{\sim}{B}^{(E)}_{\ell m}$$

and

$$i\,\underset{\sim}{\nabla} \times \underset{\sim}{L} = \underset{\sim}{r}\,\nabla^2 - \underset{\sim}{\nabla}\left(1 + r\,\frac{\partial}{\partial r}\right),$$

and rewriting the last $\underset{\sim}{\nabla}$ as

= (unit vector in r-direction) times $(\partial/\partial r)$ plus a tangential operator $\underset{\sim}{\nabla}_{\text{tang}}$ that <u>commutes with all functions of r</u>,

(a) Derive an expression in terms of $f_1(kr)$ and the derivative of $f_1(kr)$ for the strength of the <u>tangential component of the electric field ONLY</u> in its dependence upon r, overlooking all questions of the directionality of this field in θ, φ and overlooking also all dependence upon θ and φ.

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working space for III (a) continued

(b) What boundary conditions must $f_\ell(kr)$ satisfy on the surface of the sphere and at infinity, and why? And therefore by which kind of spherical Bessel function (j_ℓ, n_ℓ, $h_\ell^{(1)}$, $h_\ell^{(2)}$) must $f_\ell(kr)$ be represented?

(c) Given the formulas

$$j_3(x) = \left(\frac{15}{x^4} - \frac{6}{x^2}\right)\sin x - \left(\frac{15}{x^3} - \frac{1}{x}\right)\cos x$$

$$n_3(x) = -\left(\frac{15}{x^4} - \frac{6}{x^2}\right)\cos x - \left(\frac{15}{x^3} - \frac{1}{x}\right)\sin x$$

$$h_3^{(1)}(x) = \frac{e^{ix}}{x}\left(1 + \frac{6i}{x} - \frac{15}{x^2} - \frac{15i}{x^3}\right)$$

and given that the relevant one of the following expressions has roots as indicated:

$u^2 + 3u + 3$: $u = -1.50 \pm i\,0.866$

$u^3 + 6u^2 + 15u + 15$: $u = -3.32$; $u = -1.84 \pm i\,1.75$

$u^3 + 3u^2 + 6u + 6$: $u = -1.60$; $u = -0.702 \pm i\,1.81$

$u^4 + 6u^3 + 21u^2 + 45u + 45$: $u = -2.16 \pm i\,0.871$; $u = -0.843 \pm i\,2.76$

express the quantities ω_0 and Γ in terms of c and a and the appropriate pure numbers picked out of these factors. Explain your reasoning and why you exclude one "answer" in favor of another:

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working space for III. (c)

IV. POTPOURRI

Answer all three parts (a, b, c).

(a) Hollow closed cavity A has a conducting wall and in it there is a standing electromagnetic wave with a certain root mean square EMF which is dying out slowly with time because the wall is not a perfect conductor. Cavity B and the wave in it are identical in every respect except that the material of the wall of B has four times the conductivity of the wall of A. In which are the surface currents greatest? Why? In which are the I^2R losses greatest? Why? In which is the damping rate highest? Why? Be as quantitative as you can.

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IV. (a) working space continued

(b) Compare and contrast the TE and TM modes of emission of a pure outgoing electromagnetic wave from the surface of a perfectly conducting sphere as regards (i) charge induced on the surface, (ii) boundary conditions on the radial factor $f(kr)$ [or $g_l(kr)$] in the solution at the surface, (iii) boundary conditions at infinity, (iv) place of storage of the energy about to be emitted, (v) kind of source which, if suddenly "unfrozen", would give rise to these two types of radiation:

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(c) Discuss qualitatively but with the help of at least 3 large, clear, well labelled diagrams of the particle's track in space the response of a charged particle, originally at rest, to a plane monochromatic electromagnetic wave in three levels of approximation: first considering only the action of the electric field; then also the action of the magnetic field; and finally the action of mean radiation pressure (that it experiences as a result of scattering into all directions radiation that is incident from only one direction).

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working space for IV. (c) continued

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V. VARIATIONAL PRINCIPLE

Compare and contrast the variational principles for (i) a charged particle moving at sometimes relativistic speeds from one assigned x_1, y_1, z_1 at t_1 to preassigned x_2, y_2, z_2, at t_2 under the influence of a field described by a preassigned electromagnetic 4-potential and (ii) the electromagnetic field varying with time in charge-free-space in such a way as to get from preassigned magnetic field configuration $B^{(1)}(x, y, z)$ at time t_1 to preassigned magnetic field configuration $B^{(2)}(x, y, z)$ at time t_2, under these heads:

(a) Compare and contrast the "action integrals" that are to be extremized in the two cases.

(b) Is one fixing "dynamic coordinates" or fixing "dynamic momenta" at the two limits?

(c) Compare and contrast: Are the quantities that are to be varied to achieve the extremum physically well defined? Explain!

(d) Compare and contrast: Relations between the Euler-Lagrange equations that say that these equations are or are not independent of one another? Explain.

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(e) Compare and contrast the concepts of total system energy and total system momentum that are forced on one in the two very different cases of (i) the particle and (ii) the electromagnetic field in charge-free space.